

## NUMERICAL PROBLEMS CH#8

P 8.1 :- The wavelength of the signals from a radio transmitter is 1500m and the frequency is 200 KHz. What is the wavelength for a transmitter operating at 1000 KHz and with what speed the radio waves travel

#SOLUTION# :- DATA :-

$$\lambda = 1500\text{m}$$

$$f = 200\text{ KHz} = 200 \times 10^3\text{ Hz}$$

$$f' = 1000\text{ KHz} = 1000 \times 10^3\text{ Hz}$$

$$v = ?$$

$$\lambda' = ?$$

$$\therefore v = f \lambda$$

$$v = 200 \times 10^3 \times 1500$$

$$v = 3 \times 10^8\text{ m s}^{-1}$$

For determining the wavelength of 2nd signal

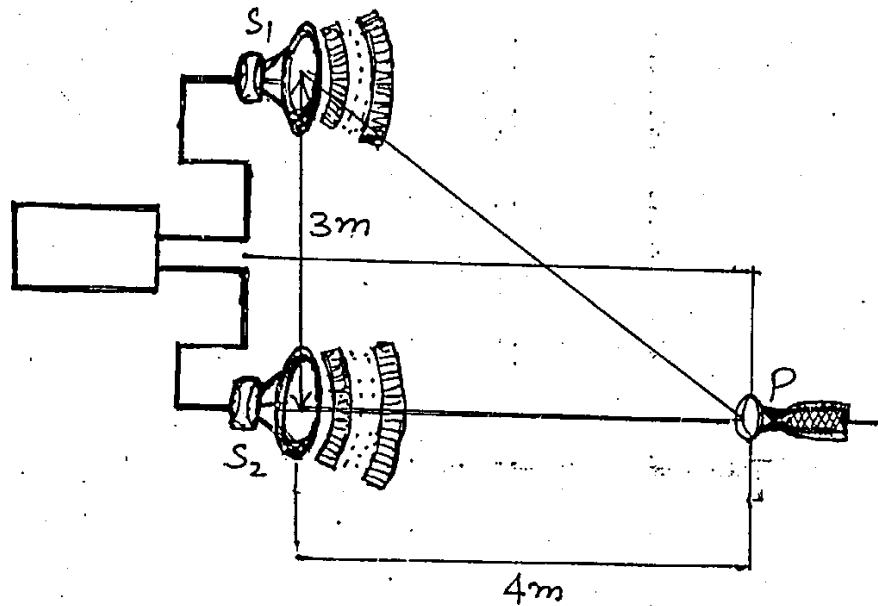
$$v = f' \lambda'$$

$$\Rightarrow \lambda' = \frac{v}{f'} = \frac{3 \times 10^8}{1000 \times 10^3}$$

$$\lambda' = 300\text{m}$$

P 8.2 :- Two speakers are arranged as shown in the fig. The distance between them is 3m and they emit a constant tone of 344 Hz. A microphone p is moved along a line parallel to and 4m from the

line connecting the two speakers. It is found that the tone of max. loudness is heard and displayed on the CRO when the microphone is on the centre of the line and directly opposite each speaker. Calculate the speed of sound.



∴ DATA :-

Distance between the speakers =  $S_1S_2 = 3\text{m}$

Distance b/w speaker and microphone  
=  $S_2P = 4\text{m}$

$f = 344\text{Hz}$      $S_1P = ?$

$v = ?$

For producing loud or maximum sound, the condition for constructive interference is that the path difference should be equal to integral multiple of wave length. At point P,  
path difference =  $S_1P - S_2P = \lambda$ .

For  $S_1P$  using  $\Delta S_1S_2P$  where

$$(S_1P)^2 = (S_1S_2)^2 + (S_2P)^2$$

$$(S_1P)^2 = (3)^2 + (4)^2$$

$$S_1P = \sqrt{9+16} = 5m$$

So  $\lambda = S_1P - S_2P$

$$\lambda = 5 - 4 = 1m$$

$$\begin{aligned} \therefore v &= f\lambda \\ &= 344 \times 1 \\ &= 344 \text{ m s}^{-1} \end{aligned}$$

P 8.3 :- A stationary wave is established in a string which is 120cm long and fixed at both ends. The string vibrates in four segments, at a frequency of 120 Hz. Determine its wavelength and fundamental frequency.

# SOLUTION # :- DATA :-

$$l = 120 \text{ cm} = 1.2 \text{ m}$$

$$f_4 = 120 \text{ Hz}$$

$$n = 4$$

$$\lambda_4 = ?$$

$$f_1 = ?$$

From general equation of wavelength

$$\lambda_n = \frac{2l}{n}$$

$$\Rightarrow \lambda_4 = \frac{2l}{4} = \frac{l}{2} = \frac{1.2}{2}$$

$$\lambda_4 = 0.6 \text{ m}$$

Also  $f_n = n f_1$

$$\Rightarrow f_1 = \frac{f_n}{n}$$

$$f_1 = \frac{f_4}{4} = \frac{120}{4}$$

$$f_1 = 30 \text{ Hz}$$

P 8.4 The frequency of a note emitted from a stretched string is 300 Hz. what will be the frequency of this note when  
 (a) the length of the wave is reduced by one third without changing the tension.  
 (b) the tension is increased by one third without changing the length of the wire.

-: SOLUTION :- DATA :-

$$f = 300 \text{ Hz}$$

$$(a) \quad f' = ?$$

$$(b) \quad f'' = ?$$

(a) when the wavelength is reduced by  $\frac{1}{3}$  of its original value, the new wavelength is given as.

$$\lambda' = \lambda - \frac{\lambda}{3} = \frac{3\lambda - \lambda}{3} = \frac{2\lambda}{3}$$

As the tension is constant, hence the velocity of the wave will not change.

$$\therefore v' = v$$

$$f' \lambda' = f \lambda$$

$$f' = \frac{f \lambda}{\lambda'}$$

$$= \frac{300 \times \lambda}{2\lambda/3} = \frac{300 \times 3}{2} = 450$$

$$f' = 450 \text{ Hz}$$

(b) As the equation for fundamental frequency is given as

$$f = \frac{1}{2l} \sqrt{\frac{F}{m}} \quad \text{--- (A)}$$

As the tension is increased by  $\frac{1}{3}$  of its original value, which is given as

$$\begin{aligned} F'' &= F + \frac{F}{3} \\ &= \frac{3F + F}{3} \\ F'' &= \frac{4F}{3} \end{aligned}$$

So the new frequency is given as

$$f'' = \frac{1}{2l} \sqrt{\frac{F''}{m}}$$

$$f'' = \frac{1}{2l} \sqrt{\frac{4F}{3m}} \quad \text{--- (B)}$$

Dividing equation (B) by equation (A)

$$\frac{f''}{f} = \frac{\frac{1}{2l} \sqrt{\frac{4F}{3m}}}{\frac{1}{2l} \sqrt{\frac{F}{m}}}$$

$$\frac{f''}{f} = \frac{1}{2l} \sqrt{\frac{4}{3}} \sqrt{\frac{F}{m}} \times \frac{2l \sqrt{\frac{m}{F}}}{1}$$

$$\frac{f''}{f} = \sqrt{\frac{4}{3}}$$

$$f'' = \sqrt{\frac{4}{3}} f$$

$$= \sqrt{1.33} f$$

$$= 1.153 \times 300 = 345.9 \text{ Hz}$$

$$f'' \approx 346 \text{ Hz}$$

P 8.5 :- An organ pipe has a length of 50cm. Find the frequency of fundamental note and the next harmonic when it is

(a) open at both ends  
 (b) closed at one end

Speed of sound =  $350 \text{ m s}^{-1}$

#SOLUTION#

DATA:-

$l = 50 \text{ cm} = 0.5 \text{ m}$

$v = 350 \text{ m s}^{-1}$

(a)  $f_1 = ?$      $f_2 = ?$

(b)  $f'_1 = ?$      $f'_3 = ?$

(a) For open organ pipe

$f_n = n f_1$

but  $f_1 = \frac{v}{2l} = \frac{350}{2 \times 0.5}$

$f_1 = 350 \text{ Hz}$

$\therefore f_2 = 2 f_1$   
 $= 2 \times 350 = 700 \text{ Hz}$

(b) For closed organ pipe

The fundamental frequency in case of closed organ pipe is

$f'_1 = \frac{v}{4l} = \frac{350}{4 \times 0.5}$

$f'_1 = 175 \text{ Hz}$

Hence  $f'_3 = 3 f'_1$

$= 3 \times 175$

$f'_3 = 525 \text{ Hz}$

only odd harmonics are existed in closed organ pipe.

P 8.6: A church organ consists of pipes, each open at one end, of different lengths. The minimum length is 30mm and the longest is 4m. Calculate the frequency range of the fundamental notes.

$$\text{Speed of sound} = 340 \text{ m s}^{-1}$$

# SOLUTION # :- DATA :-

$$v = 340 \text{ m s}^{-1}$$

$$\text{mini. length} = l = 30 \text{ mm} = 0.03 \text{ m}$$

$$\text{Max. length} = l' = 4 \text{ m}$$

$$f = ?$$

$$f' = ?$$

In closed organ pipe

$$\therefore f = \frac{v}{4l} = \frac{340}{4 \times 0.03} = \frac{340}{0.12} \\ = 2833.33 \text{ Hz} \approx 2833 \text{ Hz}$$

$$\text{Also } f' = \frac{v}{4l'} = \frac{340}{4 \times 4} = \frac{340}{16} \\ = 21.25 \text{ Hz}$$

$$\approx 21 \text{ Hz}$$

Hence the frequency range is from 21 Hz to 2833 Hz.

P 8.7 :- Two tuning forks exhibit beats at a beat frequency of 3 Hz. The frequency of one fork is 256 Hz. Its frequency is lowered slightly by adding a bit of wax to one of its prong. The two forks then exhibit a beat frequency of 1 Hz. Determine the frequency of second tuning fork.

# SOLUTION # DATA:-

Beat frequency before loading =  $n = 3$ Beat frequency after loading =  $n' = 1$ 

$$f_A = 256 \text{ Hz}$$

$$f_B = ?$$

Before loading

$$f_B - f_A = \pm n$$

$$\Rightarrow f_B = f_A \pm n$$

$$= 256 \pm 3$$

$$= 259 \text{ Hz}, 253 \text{ Hz}$$

After loading

$$f_B - f_A = \pm n'$$

$$f_B = f_A \pm n'$$

$$= 256 \pm 1$$

$$= 257 \text{ Hz}, 255 \text{ Hz}$$

Let 259 Hz be the exact (correct)

frequency of tuning fork B. when tuning fork A is loaded slightly, its frequency starts decreasing say 255, 254, 253.....

and beat frequency goes on increasing and will never be equal to 1, Hence

our supposition is wrong. So 253 Hz is the exact frequency of tuning fork B. Now when

tuning fork A is loaded slightly, its

frequency is decreased. Let it is decreased from 256 to 254 Hz. Then only one beat



is produced because the frequency difference b/w the two tuning forks is equal to 1 which satisfies the condition.

P 8.8 :- Two cars P and Q are travelling along a motorway in same direction. The leading car P travels at a steady speed of  $12 \text{ m s}^{-1}$ , the other car Q travelling at a steady speed of  $20 \text{ m s}^{-1}$ , sound its horn to emit a steady note which P's driver estimates, has a frequency of  $830 \text{ Hz}$ . What frequency does Q's own driver hear?

$$\text{Speed of sound} = 340 \text{ m s}^{-1}$$

# SOLUTION # :- DATA :-

$$\text{Speed of Car P} = 12 \text{ m s}^{-1}$$

$$\text{Speed of Car Q (Source)} = 20 \text{ m s}^{-1}$$

$$\text{Relative speed} = v_s = 20 - 12 = 8 \text{ m s}^{-1}$$

$$f' = 830 \text{ Hz}$$

$$v = 340 \text{ m s}^{-1}$$

$$f_q = ?$$

As the source is approaching the listener

Hence

$$f' = \left( \frac{v}{v - v_s} \right) f_q$$

$$f_q = \frac{(v - v_s) f'}{v}$$

$$= \frac{340 - 8}{340} \times 830$$

$$f_q = 810.47 \text{ Hz} \approx 810 \text{ Hz}$$

P 8.9 - A train sounds its horn before it sets off from the station and an observer waiting on the platform estimates its frequency at  $1200\text{ Hz}$ . The train then moves off and accelerates steadily. Fifty seconds after departure, the driver sounds the horn again and the platform observer estimates the frequency at  $1140\text{ Hz}$ . Calculate the train speed  $50\text{ s}$  after departure. How far from the station is the train after  $50\text{ s}$ ?

# SOLUTION #-: DATA:- (Speed of sound =  $340\text{ m s}^{-1}$ )

$$f = 1200\text{ Hz}$$

$$f' = 1140\text{ Hz}$$

$$v = 340\text{ m s}^{-1}$$

$$v_s = ?$$

$$t = 50\text{ s}$$

$$s = ?$$

As the source (train) is moving away from the listener at the platform

$$\therefore f' = \left( \frac{v}{v + v_s} \right) f$$

$$1140 = \frac{340}{340 + v_s} \times 1200$$

$$340 + v_s = \frac{340 \times 1200}{1140}$$

$$v_{av} = \frac{v_1 + v_2}{2}$$

$$= \frac{0 + 17.9}{2}$$

$$= 8.95\text{ m/s}$$

$$v_s = \frac{40800}{1140} - 340$$

$$= 17.89\text{ m s}^{-1} \approx 17.9\text{ m s}^{-1}$$

$$\text{Also } s = v_{av} t \Rightarrow s = 8.95 \times 50$$

$$s = 447.5\text{ m} \approx 448\text{ m}$$

P 8.10:- The absorption spectrum of faint galaxy is measured and the wavelength of one of the lines identified as the Calcium  $\alpha$  line is found to be 478 nm. The same line has a wavelength of 397 nm when measured in the laboratory.

- (a) Is the galaxy moving towards or away from the earth?  
 (b) Calculate the speed of the galaxy relative to earth. (speed of light =  $3.0 \times 10^8 \text{ m s}^{-1}$ )

# SOLUTION # DATA:-

$$\text{Apparent wavelength} = \lambda' = 478 \text{ nm} \\ = 478 \times 10^{-9} \text{ m}$$

$$\text{Actual Wavelength} = \lambda = 397 \text{ nm} \\ = 397 \times 10^{-9} \text{ m}$$

$$\text{Speed of light} = c = 3 \times 10^8 \text{ m s}^{-1}$$

$$f' = ?$$

$$f = ?$$

$$v_s = ?$$

$$\therefore c = f \lambda$$

$$\Rightarrow f = \frac{c}{\lambda} = \frac{3 \times 10^8}{397 \times 10^{-9}}$$

$$f = 7.56 \times 10^{14} \text{ Hz}$$

$$\text{Also } f' = \frac{c}{\lambda'} = \frac{3 \times 10^8}{478 \times 10^{-9}} \\ = 6.28 \times 10^{14} \text{ Hz}$$

As apparent frequency  $f'$  is less than actual frequency  $f$ , thus the galaxy is

moving away from the earth.

For determining the speed of the galaxy moving away from the earth, we use the following equation.

$$f' = \left( \frac{c}{c + v_s} \right) f$$

By rearranging  $c f$

$$\Rightarrow c + v_s = \frac{c f}{f'}$$

$$v_s = \frac{c f}{f'} - c$$

$$= \frac{3 \times 10^8 \times 7.56 \times 10^{14}}{6.28 \times 10^{14}} - 3 \times 10^8$$

$$= \frac{22.68 \times 10^8}{6.28} - 3 \times 10^8$$

$$= 3.611 \times 10^8 - 3 \times 10^8$$

$$= (3.611 - 3) \times 10^8 = .611 \times 10^8$$

$$= 6.11 \times 10^7 \text{ m/sec.}$$

# THE END #

fingers along the neck there by changing the length of vibrating string.

## TRANSVERSE STATIONARY WAVES

### IN AIR COLUMNS

Stationary waves can also be set up by vibrating air column in a device called organ pipe.

Here two conditions are possible

1- When reflecting end is open.

2- When reflecting end is closed.

#### 1:- REFLECTING END IS OPEN :-

If reflecting end of the pipe is open then it is called open organ pipe. As both the ends are open hence air molecules vibrate with maximum amplitudes thus both the ends act as antinodes. Longitudinal waves set up inside the air column enclosed inside the pipe are represented by transverse curved lines which show the variation of amplitude of air particles along the axis of the pipe.

Now we discuss the different modes of vibrating air column in open organ pipe.

#### :- 1ST MODE OF VIBRATION :-

In this mode of vibration a node is formed at the centre between the two antinodes at the ends. as shown in the figure.