L.C.M. and H.C.F. of Factions.

L.C.M. of two or more fractions

$$= \frac{\text{L.C.M. of numerators}}{\text{H.C.F. of denominators}}$$

H.C.F. of two or more fractions

$$= \frac{\text{L.C.M. of numerators}}{\text{H.C.F. of denominators}}$$

Model Examples

Q1. The H.C.F. of two numbers is 34 and their L.C.M. is 4284. If one of the numbers is 204, find the other.

Solution: As product of 2 numbers = their H.C.F. × L.C.M.

The other number is $= \frac{34 \times 4284}{204}$

$= 714$ **Ans.**

Q2. What is the highest number of four digits which will leave a remainder of 1 when divided by any of numbers 6, 9, 12, 15, or 18?

Solution: L.C.M. of 6, 9, 12, 15, 18 = 180

Greatest no. of 4 digits = 9999

Greatest no. of 4 digits divisible by

$180 = 9999 - 99 = 9900$

$$\begin{array}{r} 55 \\ 180\overline{)9999} \\ \underline{900} \\ 999 \\ \underline{900} \\ 99 \end{array}$$

$\therefore$ Reqd. No. $= 9900 + 1 = 9901$ **Ans.**

Q3. Three men A, B and C go walking round a circle one mile in circumference at the rates of 160, 120 and 105 yards per minute, respectively. If they all start together and walk in the same direction, when will they first be together again?

Solution: Circumference of the circle = 1 mile or 1760 yds.

A will complete the circle in

$= \frac{1760}{160} = 11$ min.

B will complete the circle in

$= \frac{1760}{120} = \frac{44}{3}$ min.

C will complete the circle in

$= \frac{1760}{105} = \frac{352}{21}$ min.

L.C.M. of 11, $\frac{44}{3}, \frac{352}{21} = 352$ minutes.

i.e., they will be together again first after 352 min. or 5 hrs. 52 min. **Ans.**

Multiple Choice Questions (MCQs)

Q1. A neon sign flashes every 3 seconds, another sign flashes every 5 seconds, and a third flashes every 7 seconds. If they all flash together, how many seconds will pass before they all flash simultaneously again?

(A) 15 seconds **(B)** 35 seconds

(C) 105 seconds **(D)** 21 seconds

Q2. The greatest number which exactly divides 1155 and 735 is:

(A) 25 **(B)** 5

(C) 15 (D) 105

Q3. The least number which when divided by 35, leaves remainder of 25; when divided by 45 leaves a remainder of 35 and when divided by 55 leaves 45 as remainder, is:

(A) 3455 (B) 3465
(C) 3475 (D) 10

Q4. The L.C.M of 12,20,24,32 is:

(A) 240 (B) 360
(C) 480 (D) 600

Q5. How many whole bricks $6 \times 12 \times 24$ cm will be sufficient to construct a solid cube of minimum size?

(A) 4 (B) 6
(C) 8 (D) 12

Q6. If the L.C.M and H.C.F of two numbers are 150 and 30 respectively, and one of the numbers is 18, find the other number?

(A) 250 (B) 180
(C) 150 (D) 170

Q7. The product of two numbers is 2500. If their L.C.M is 125, then their H.C.F is:

(A) 20 (B) 250
(C) 125 (D) None of these

Q8. It takes Riaz 30 minutes to mark a paper. Razi only need 25 minutes to mark a paper. If they both start marking papers at 11 : 00 AM, what is the first time they will finish marking a paper at the same time?

(A) 12 : 30 (B) 12 : 45
(C) 1 : 30 (D) 12 : 25

Q9. Sonia buys two off-cuts of ribbon in a sale. One is 153 cm long. The other is 204 cm long. She cuts them so that she ends up with a number of pieces all the same length. What is the greatest length each piece can be?

(A) 39 (B) 6
(C) 17 (D) 51

Q10. A farmer wants to fence a triangular field. He plans to put a fencing post in each corner and place other posts at equal distance along its sides. He wants the posts to be as far apart as possible. The sides of the field are 477 feet 2412 feet and 639 feet long. How far apart will the posts be?

(A) 18 feet (B) 9 feet
(C) 27 feet (D) 159 feet

Q11. Find the greatest number of 4 digits which when divided by 18, 24, 30 and 36 leaves a remainder 17 in each case.

(A) 360 (B) 9360
(C) 3600 (D) 9377

Q12. The least number which when divided by 12, 15 and 18 leaves 5 as remainder in each case is:

(A) 180 (B) 175
(C) 185 (D) 125

Q13. The greatest number which divides 2400 and 3600 leaving 48 and 60 respectively, as remainder is:

(A) 9 (B) 7
(C) 17 (D) 10

Q14. Ahmed has a rectangular garden measuring 4.32m by 3.36m. He wants to divide it into square plots of equal size. What is largest sized square he can use?

(A) 0.24 (B) $\sqrt{3}$
(C) 0.48m (D) 0.16

Q15. The chairs in the school hall can be set out in 35 equal rows or in 45 equal rows or in 105 equal rows are:

(A) 600 (B) 400

Chapter 3

HIGHEST COMMON FACTOR & LEAST COMMON MULTIPLE

The highest common factor of two or more numbers is the greatest number which divides each of them exactly.

Methods of finding H.C.F.

(i) ***By Prime Factors.***

Resolve the given number into their prime factors. The product of all prime common factors is known as H.C.F.

Model Example

Find the H.C.F. of 630, 1050 and 1260.

Solution: $630 = 2.3.3.5.7$

$1050 = 2.3.5.5.7$

$1260 = 2.2.3.3.5.7$

$\therefore$ H.C.F. is $2.3.5.7 = 210$. **Ans.**

(ii) ***By Division*:**

Find the H.C.F. of 5133 and 3953

$$
\begin{array}{r}
3953\overline{)\,5133\,}(1 \\
\underline{3953} \\
1180\overline{)\,3953}(3 \\
\underline{3540} \\
413\overline{)\,1180}(2 \\
\underline{826} \\
354\overline{)\,413}(1 \\
\underline{354} \\
59\overline{)\,354}(6 \\
\underline{354} \\
\times
\end{array}
$$

Various Steps:

Step I. Dividing the greatest number by the lesser, we get the remainder 1180.

Step II. Dividing the previous divisor 3953 by 1180, we get the remainder 413.

Step III. Dividing the previous divisor 1180 by 413 we get the remainder 354.

Step IV. Dividing the previous divisor 413 by 354 we get the remainder 59.

Step V. Dividing the previous divisor 354 by 59 we get no remainder.

$\therefore$ The last divisor 59 is the H.C.F.

H.C.F. is also known as Greatest Common Measure (G.C.M.)

LEAST COMMON MULTIPLE (L.C.M)

The Least Common Multiple of two or more given numbers is the least number which is exactly divisible by each of them.

Methods of Finding L.C.M.

(i) By Factors. Resolve the given numbers into prime factors, and find the product of the highest powers of all the factors that occur in the given number. The product will be the required L.C.M.

Model Example

Q1. Find the L.C.M. of 70, 80, 90.

Solution: $70 = 2 \times 5 \times 7$

$80 = 2^4 \times 5$

$90 = 2 \times 3^2 \times 5$

L.C.M. $= 2^4.3^2.5.7 = 5040$ Ans.

(ii) ***With the help of H.C.F.*** The product of two numbers is equal to the product of their L.C.M. and H.C.F.

$\therefore$ L.C.M. of two numbers

$$= \frac{\text{Product of numbers}}{\text{H.C.F.}}$$

(C) 40 (D) 80

Q16. **Three bells toll after intervals of 6, 9 and 15 minutes, respectively. If they toll together at 5 p.m when will they toll together next?**

(A) 6 : 30 (B) 5 : 30

(C) 6 : 45 (D) 5 : 45

Explanatory Answers

Q1. (C) The L.C.M of 3,5 and 7 will give the answer

3	3 – 5 – 7
5	1 – 5 – 7
7	1 – 1 – 7
	1 – 1 – 1

$= 3 \times 5 \times 7 = 105$

Q2. (D) The required number is the H.C.F of 1155 and 735

$$735 \overline{)1155}\ (1$$
$$420 \overline{)735}\ (1$$
$$315 \overline{)420}\ (1$$
$$105 \overline{)315}\ (3$$
$$315$$
$$\times$$

The greatest number required is 105.

Q3. (A) The least number which is completely divided by 35, 45 and 55, is their L.C.M. which is 346: We want to find the least number which on dividing by 35, 45 and 55 leave remainders 25, 3 and 45 respectively *i.e.*, 10 less than the quotient in each case. Hence such a number is 3465 10 = 3455

Q4. (C)

2	12 – 20 – 24 – 32
2	6 – 10 – 12 – 16
2	3 – 5 – 6 – 8
2	3 – 5 – 3 – 4
2	3 – 5 – 3 – 2
3	3 – 5 – 3 – 1
5	1 – 5 – 1 – 1
	1 – 1 – 1 – 1

The L.C.M. of 12, 20, 24 and 32 is

$2^5 \times 3 \times 5 = 32 \times 3 \times 5 = 480$

Q5. (C) One edge of the minimum cube must be 24 cms, the least common multiple of 6, 12 and 24 Thus, it will have a volume of 24 × 24 × 24 cubic centimeters which is equal to 8 bricks

i.e., $\frac{24 \times 24 \times 24}{6 \times 12 \times 24} = 8$

Q6. (A) Product of two numbers = L.C.M × H.C.F

$18 \times$ 2nd number $= 150 \times 30$

$$\text{2nd number} = \frac{150 \times 30}{18}$$

$= 250$

Q7. (A) Product of two numbers = L.C.M × H.C.F

$2500 = 125 \times$ H.C.F

$$\Rightarrow \quad \text{H.C.F} = \frac{2500}{125}$$

$= 20$

Q8. (C) The question asks for the first time they will finish at the same time. So, we must find least common multiple

5	25 – 30
5	5 – 6
6	1 – 6
	1 – 1

$6 \times 5 \times 5 = 150$ minutes

$= 2 : 30$ hours

So they will finish marking at 1 : 30 PM.

Q9. (D) The HCF of 153 and 204 gives the wanted length

3	153
3	51
17	17
	1

2	204
2	102
3	51
17	17
	1

$153 = 3 \times ③ \times \boxed{17}$

$204 = 2 \times 2 \times ③ \times \boxed{17}$

$HCF = 17 \times 3 = 51$

Greatest length $= 51$cm

Q10. (B) The HCF of 477, 2412 and 639 gives the wanted length.

3	477
3	159
	53

2	2412
2	1206
3	603
3	201
	67

3	639
3	213
	71

$477 = ③ \times \boxed{3} \times 53$

$2412 = 2 \times 2 \times ③ \times \boxed{3} \times 67$

$639 = ③ \times \boxed{3} \times 71$

$H.C.F = 3 \times 3 = 9$

Q11. (D) The number which is divisible by 18, 24, 30 and 36 is divisible by their L.C.M

2	18 – 24 – 30 – 36
2	9 – 12 – 15 – 18
3	9 – 6 – 15 – 9
3	3 – 2 – 5 – 3
2	1 – 2 – 5 – 1
5	1 – 1 – 5 – 1
	1 – 1 – 1 – 1

$\therefore$ L.C.M $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$

The greatest number of 4–digits is 9999. Now we find the greatest multiple of 360 less than 9999.

$$360 \overline{)\,9999\,(\,26}$$
$$720$$
$$2799$$
$$2160$$
$$639$$

Thus $9999 - 639 = 9360$ is exactly divisible by 360. But the required number leaves a remainder of 17 in each case. Hence, the number is

$9360 + 17 = 9377$

Q12. (C) Required number = L.C.M of 12, 15, 18

2	12, 15, 18
3	6 – 15 – 9
2	2 – 5 – 3
3	1 – 5 – 3
5	1 – 5 – 1
	1 – 1 – 1

∴ L.C.M $= 2 \times 2 \times 3 \times 3 \times 5 = 180$

The required least number $= 180 + 5 = 185$

Q13. **(D)** As 48 and 60 remainders when 2400 and 3600 are divided by the numbers $2400 - 48 = 2352$ and

$3600 - 60 = 3540$ must be exactly divisible by the number.

The H.C.F of 2352 and 3540 is the required number.

$$2352 \overline{)3540}\ (1$$
$$2350\ \overline{)2352}\ (1$$
$$1190\ \overline{)2350}\ (1$$
$$1160\ \overline{)1190}\ (1$$
$$30\ \overline{)1160}\ (38$$
$$90$$
$$260$$
$$240\ \overline{)30}\ (1$$
$$20\ \overline{)20}\ (2$$
$$20$$
$$\times$$

a. The H.C.F of 2350 and 3540 is 10.

b. The required greatest number is 10.

Q14. **(C)**

$$3.36 \overline{)4.32}\ (1$$
$$.96\ \overline{)3.36}\ (3.5$$
$$3.36$$
$$\times$$

$0.96 \div 2 = 0.48\text{m}$

Q15. **(A)**

5	25 – 40 – 120
5	5 – 8 – 24
8	1 – 8 – 24
3	1 – 1 – 3
	1 – 1 – 1

$= 5 \times 5 \times 8 \times 3 = 600$ chairs

Q16. **(A)**

3	6 – 9 – 15
2	2 – 3 – 5
3	1 – 3 – 5
5	1 – 1 – 5
	1 – 1 – 1

L.C.M of 6, 9 and 15 $= 3 \times 2 \times 3 \times 5$

$= 90$

∴ The bells will toll after 90 minutes, it mean at 6 : 30.