Chapter 2

MULTIPLICATION AND DIVISION

MULTIPLICATION

Multiplication is a short method of adding the same number repeatedly.

PROPERTIES OF MULTIPLICATION

1. Multiplication is commutative for rational numbers.
   Example:
   $$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} = \frac{c}{d} \times \frac{a}{b}$$
   $$\frac{2}{3} \times \frac{5}{7} = \frac{10}{21} = \frac{5}{7} \times \frac{2}{3}$$

2. Multiplication is associative for rational numbers.
   Example:
   $$\frac{a}{b} \times \left(\frac{c}{d} \times \frac{e}{f}\right) = \frac{ace}{bdf} = \left(\frac{a}{b} \times \frac{c}{d}\right) \times \frac{e}{f}$$
   $$\frac{2}{3} \times \left(\frac{5}{7} \times \frac{11}{13}\right) = \frac{110}{273} = \left(\frac{2}{3} \times \frac{5}{7}\right) \times \frac{11}{13}$$

3. Multiplication is distributive over addition and subtraction for rational numbers.
   Example:
   $$\frac{a}{b} \times \left(\frac{c}{d} \pm \frac{e}{f}\right) = \frac{a}{b} \times \frac{c}{d} \pm \frac{a}{b} \times \frac{e}{f}$$
   $$\frac{2}{3} \times \left(\frac{5}{7} \pm \frac{11}{13}\right) = \frac{2}{3} \times \frac{5}{7} \pm \frac{2}{3} \times \frac{11}{13}$$

4. For any rational number $\frac{x}{y}, \frac{x}{y} \times 1 = \frac{x}{y} = 1 \times \frac{x}{y}$, one is called multiplicative identity.

5. Two rational numbers $\frac{a}{b}$ and $\frac{b}{a}$ are the multiplicative inverses of each other.
   $$\frac{a}{b} \times \frac{b}{a} = 1 = \frac{b}{a} \times \frac{a}{b}$$

Note: The sign of the product is +ive, if there are an even number of negative factors or there are n(negative factors. The sign of the product is –ive, if there are an odd number of negative factors

DIVISION

The process of subtraction of the same number form a given number for a few times is called division (÷), *i.e.*
$$6 \div 2 = 3$$
(2 can be subtracted 3 times from 6)

IMPORTANT POINTS

1. Division is the inverse operation of multiplication. For example $6 \div 2 = 3$ means to find the number b; which 2 should be multiplied so as to obtain 6.
   Because $\quad 3 \times 2 = 6$
   Therefore, $\quad 6 \div 2 = 3$

2. When a number is divided by another number, the first number *i.e.* the number which is being divide is called the *dividend*, the second number which divides is called the *divisor* and the number obtaine as a result of division is called the *quotient*. In the above example, 6 is the dividend, 2 is the diviso and 3 is the quotient.

3. The operation of division starts from the left whereas the operations of addition, subtraction an multiplication start from the right.

Divisibility:

The following table gives the rules to test the divisibility from 2 to 19.

Divisibility by	If	Example
2	Any number in the unit's place which is either even or zero.	12, 10, 26, 32, 38, 567992, 11110234
3	The sum of digits is divisible by 3.	321 : 3 + 2 + 1 = 6 is divisible by 3.
4	The last two digits of a number is divisible by 4.	725324 : 24 is divisible by 4.
5	The number ends with 5 or zero.	4112370, 5321095, 3331210, etc.
6	A number is divisible by 2 and the sum of the digits of the number is multiple of 3.	342, 63924, 154, 261 etc.
8	The last three digits of a number is divisible by 8. or The last three digits of a number are zero.	2125000, 135923120, 7792320, 1256, etc.
9	The sum of all the digits of a number is divisible by 9.	33456735 : 3 + 3 + 4 + 5 + 6 + 7 + 3 + 5 = 36 divisible by 9.
10	Any number which ends with zero.	70, 789790, 7111130, 5773313570, 112300100 etc.
11	The sum of digits at odd and even places are respectively equal or differ by a number divisible by 11.	4235682 : Sum 1 = 4 + 3 + 6 + 2 = 15 Sum 2 = 2 + 5 + 8 = 15 Sum 1 = Sum 2, the number is divisible by 11. or 283712 : Sum 1 = 2 + 3 + 1 = 6 and Sum 2 = 8 + 7 + 2 = 17, their differ 17 − 6 = 11 is divisible by 1.
12	The number which is divisible by both 4 and 3.	135792 etc.
14	The number which is divisible by both 2 and 7.	98, 504 etc.
15	The number which is divisible by 3 and 5.	360, 733352215 etc.
16	The number whose last 4 digit number is divisible by 16.	253421020, 27954204 etc.
18	Any number which is divisible by 9 and has its last digit even (or zero).	2709360, 252630 etc.
25	The number formed by the last two digits of the number is divisible by 25.	257275, 25277750 etc.

Model Examples:

Q1. Multiply 63987 by 91763 is not more than 3 lines.

Solution:

$$
\begin{array}{r}
63986 \\
(\times)\ 91763 \\
\hline
\end{array}
$$

4031181	Multiplication by 63
447909 × ×	Multiplication by 700
5822817 × × ×	Multiplication by 91000
5871639081	

Q2. Find the number, one-sixth of which exceeds its one-ninth by 654.

Solution: Let the number be x

$$\therefore \quad \frac{x}{6} - \frac{x}{9} = 654$$

$$\frac{x}{18} = 654$$

$$\Rightarrow \quad x = 654 \times 18 = 11772 \qquad \text{Ans.}$$

Q3. Find the quotient and remainder when $x^2 + bx - 5$ is divided by $x + 1$. For what value of 'b' will the remainder be zero?

Q3. Find the quotient and remainder when $x^2 + bx - 5$ is divided by $x + 1$. For what value of 'b' will the remainder be zero?

Solution:

$$x + 1 \overline{)\, x^2 + bx - 5\,} \big(x + (b-1)$$
$$\underline{x^2 + x}$$
$$\overline{(b-1)x - 5}$$
$$\underline{(b-1)x - 1 + b}$$
$$\overline{-4 - b}$$

So Quotient $= x + b - 1$ Ans.
Remainder $= -(b + 4)$
For remainder $= 0$
$$-b - 4 = 0$$
$$\Rightarrow \quad \boxed{b = -4}$$

Q4. The speed of mail train is 1,370 meters per minute. Express it in miles per hour correct to three significant figures, given that 1 metre = 39.37 inches.

Solution: Speed of mail train =   1,370 metres per minute
$$= \quad 1370 \times 60 \text{ metres per hour}$$
$$= \quad \frac{1370 \times 60 \times 39.37}{12 \times 3 \times 1760} \text{ miles per hour}$$
$$= \quad 51.077 \text{ miles per hour} \qquad \textbf{Ans.}$$

Q5. A boy when asked to multiply a number by 7/8, divided this instead, by 7/8 and found the answer $1\frac{1}{14}$ too great. Find the number and the correct answer.

Solution: Let the number be 'x'

$\therefore$
$$\left(x \div \frac{7}{8} \right) - \left(x \times \frac{7}{8} \right) = \frac{15}{14}$$
$$\frac{8x}{7} - \frac{7x}{8} = \frac{15}{14}$$
$$\frac{64x - 49x}{56} = \frac{15}{14}$$

or
$$\frac{15x}{56} = \frac{15}{14}$$

$\therefore$
$$x = \frac{56 \times 15}{14 \times 15} = 4 \qquad \textbf{Ans.}$$

Correct answer $= 4 \times \dfrac{7}{8} = 3\dfrac{1}{2}$ **Ans.**

Q6. The sum of the squares of two consecutive integers is 1105. Find the integers and check your answer.

Solution: Let the two consecutive positive numbers be:
$$x, x + 1$$

Then sum of the squares of these consecutive numbers = 1.105

$\therefore$
$$x^2 + (x + 1)^2 = 1105$$
$$x^2 + x^2 + 2x + 1 = 1105$$
$$2x^2 + 2x - 1104 = 0$$
$$x^2 + x - 552 = 0$$

or
$$x^2 + 24x - 23x - 552 = 0$$
$$x(x + 24) - 23(x + 24) = 0$$
$$(x - 23)(x + 24) = 0$$

$\Rightarrow \qquad x = 23 \qquad \text{or } x = -24$

As the two consecutive numbers are +ve integers, therefore, we neglect the –ve number.
Thus the two consecutive numbers are 23 and 24. **Ans.**

Thus the two consecutive numbers are 23 and 24. **Ans.**