Chapter 4

SQUARE ROOT

Methods of Finding Square Root:

(*i*) **By Factors.** Resolve the number into its prime factors. The square root is the product of the prime factors taken half as many times as they occur in the number.

(*ii*) **By Division.**

Model Example

Find the square root of 2480625.

Solution:

```
   1 | 2480625   (1575
     | 1
  25 | 148
     | 125
 307 | 2306
     | 2149
3145 |  15725
     |  15725
     |      ×
```

∴   Ans. 1575

Q2. Find the square root of 43.45 to four decimal places.

Solution:

```
     6 | 43.45     (6.5916
       | 36
   125 | 7.45
       | 6.25
  1309 |  12000
       |  11781
 13181 |  21900
       |  13181
131826 |   871900
       |   790956
       |    80944
```

As remainder is more than half 6.5917   **Ans.**

Q3. Find the value of $\sqrt{\dfrac{2+\sqrt{3}}{2-\sqrt{3}}}$ correct to three decimal places.

Solution: $\sqrt{\dfrac{2+\sqrt{3}}{2-\sqrt{3}}} = \sqrt{\dfrac{(2+\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2-\sqrt{3})}}$ (Rationalization)

$$= \sqrt{\dfrac{(2+\sqrt{3})^2}{(2)^2 - (\sqrt{3})^2}}$$

$$= \sqrt{\dfrac{(2+\sqrt{3})^2}{4-3}} = 2+\sqrt{3} = 2+1.732$$

$= 3.732$ **Ans.**

Unitary Method and Chain Rule

IMPORTANCE:

The unitary method and chain rule have quite an importance in our daily life. It is explained by the following model examples.

Model Examples:

Q1. In a kilometer race A can beat B by 40 metres and B can beat C by 50 metres. How many metres can A beat C in a 500 metres race?

Solution: When A covers 1000 m.

B covers $1000 - 40$ $= 960$ m.

and   When B covers 1000 m.

C covers $1000 - 50$ $= 950$ m.

∴   When B covers 960 m.

C covers $\dfrac{950}{1000} \times 960\,m$ $= 912$ m.

i.e., when A covers 1000 m. C covers $\dfrac{912}{2}$

$= 456$ m.

or   when A covers 500 m. race, A will beat C by

$= 500 - 456 = 44$ m.

Q2. Divide Rs. 510 between A, B and C so that A gets 2/3 of what B gets and B gets ¼ of what C gets. Find the share of each.

Solution: Let C's share be Rs. x

∴   B's   //   // $= \dfrac{x}{4}$

A's   //   // $= \dfrac{2}{3} \times \dfrac{x}{4} = Rs.\ \dfrac{x}{6}$

Total amount   = Rs. 510

∴ $x + \dfrac{x}{4} + \dfrac{x}{6}$ $= 510$

$\dfrac{12x + 3x + 2x}{12}$ $= 510$

∴ $x = \dfrac{12 \times 510}{17} = 360$

∴   A's share $= \dfrac{360}{6} = Rs.\ 60$ ⎤

B's   // $= \dfrac{360}{4} = Rs.\ 90$ ⎬ **Ans.**

C's   // $= Rs.\ 360$ ⎦

Q3. Divide Rs. 600 among A, B, and C so that Rs. 40 more than 2/5 of A's share, Rs. 20 more than 2/7 of B's share, Rs. 10 more than $\dfrac{9}{17}$ of C's share may be equal.

Solution: Let $\dfrac{2}{5}$ of A's share + Rs. 40 $= \dfrac{2}{7}$ of B's share + Rs. 20 $= \dfrac{9}{17}$ of   C's share + Rs. 10 be $= x$.

∴ $\dfrac{2}{5}$ of A's share $= x - 40$

or   A's share $= \dfrac{5}{2}(x - 40)$

Similarly   B's share $= \dfrac{7}{2}(x - 20)$

$$C\text{'s share} = \frac{17}{9}(x-10)$$

As total amount, = Rs. 600

$$\therefore \quad \frac{5x-200}{2} + \frac{7x-140}{2} + \frac{17x-170}{9} = 600$$

$$\frac{45x-1800+63x-1260+34x+340}{18} = 600$$

or $\quad 142x - 3400 = 600 \times 18 = 10800$

$$142x = 10800 + 3400 = 14200$$

$$x = \frac{14200}{142} = 100$$

$\therefore \quad$ A's share $= \dfrac{5}{2}(100-40) = $ Rs. 150

$\qquad$ B's share $= \dfrac{7}{2}(100-20) = $ Rs. 280 $\Bigg\}$ Ans.

$\qquad$ C's share $= \dfrac{17}{9}(100-10) = $ Rs. 170

Q4. A garrison has enough provision for 52 days. After 20 days, a reinforcement of 400 men arrives and the food would then last for 24 days only. How many men were there in the garrison originally?

Solution: Let there be x men in the garrison originally. After 20 days no. of men $= x + 400$.
If these men had not joined, the provision would have lasted for $50 - 20 = 32$ days more.

$\therefore$ For x men the provision can last for 32 days

$\qquad // \quad 1 \quad // \qquad // \qquad // \quad 32x \quad //$

For $(x + 400)$ men of provision can last for $\dfrac{32x}{x+400}$ days

But provision lasted for 24 days

$$\therefore \quad \frac{32x}{x+400} = 24$$

or $\qquad 32x = 24x + 9600$

$\qquad\qquad 8x = 9600$

or $\qquad x = 1200$ men. **Ans.**

Multiple Choice Questions (MCQs)

Q1. What is the least positive integer which is to be added to 57592910 so that the sum may be a perfect square?

(A) 7588 $\qquad$ (B) 7

(C) 11 $\qquad$ (D) 15166

Q2. A rectangular field which is twice as long as it is broad, has an area of 14450 m², what is its perimeter?

(A) 85 m $\qquad$ (B) 510 m

(C) 165 m $\qquad$ (D) 170 m

Q3. The cost of the planting sugarcane at the rate of 6 paisa per square meter is Rs. 5840.64. What is the length of side of this square field:

(A) 312 m $\qquad$ (B) 622 m

(C) 97344 m $\qquad$ (D) 459 m

Q4. What is the smallest number which when subtracted from 1.00060219 gives a perfect square number?

(A) 0.00210 $\qquad$ (B) 210

(C)   0.00000210   (D)   0.210

Q5. The product of 313 with itself is:

(A)   97969   (B)   17.69
(C)   5536.97   (D)   195938

Q6. The size of the square which can be made using 256 square shapes with a side length of 6 cm is:

(A)   16 cm   (B)   36 cm
(C)   2.67 (approx) cm   (D)   96 cm

Q7. An instructor having 9224 students under him, arranges them into a square and finds 8 students to be excess. What is the number of students in the front row?

(A)   97   (B)   88
(C)   104   (D)   96

Q8. The difference between the first two perfect squares that end with 9 is:

(A)   11   (B)   40
(C)   30   (D)   120

Q9. A rectangular field which is 10 times as long as its breadth has an area of 75690 *sq m*. What is its perimeter?

(A)   275 m   (B)   2750 m
(C)   1914 m   (D)   191 m

Q10. A square lawn having area 0.25 *sq km* has to be enclosed with iron railings at the rate of Rs. 101-00 per metre. What will be its cost?

(A)   Rs. 20200   (B)   Rs. 2000
(C)   Rs. 202000   (D)   Rs. 100100

Q11. How many 1cm square pieces of paper can Rachael cut from a square sheet of paper with a side length of 2.1m?

(A)   22100 cm^2 (B)   44100 cm^2
(C)   27100 cm^2 (D)   36100 cm^2

Q12. If length of a rectangular field is twice that of its width. What is the perimeter of the field if its area is 1152 m^2?

(A)   256 m   (B)   144 m
(C)   24 m   (D)   96 m

Q13. $\sqrt{x^2 + y^2}$ is equal to:

(A) $(x + y)(x - y)$ (B) $\sqrt{x^2} + \sqrt{y^2}$

(C) $x + y$ (D)   None of these

Q14. $\sqrt{\dfrac{a^2}{16} + \dfrac{a^2}{25}} =$

(A) $\dfrac{a}{10}$ (B) $\dfrac{2a}{9}$

(C) $\dfrac{a\sqrt{41}}{20}$ (D) $\dfrac{41a}{20}$

Q15. The product of $\sqrt{24a}$ and $\sqrt{6a}$ is:

(A)   12a   (B) $6a^2$
(C) $12a^2$ (D) $3a\sqrt{8}$

Explanatory Answers

Q1.   **(C)**   57592910 is greater than the square of 7588 (using calculator). The next squared is the squa of 7589. $(7589)^2 = 57592921$.

Now $57592921 - 57592910 = 11$, which is the required integer to be added.

Q2.   **(B)**   As the length is twice as long as width and so its rectangle can be divided into 2 squa regions

$$\text{The area of each square region} = \frac{14450}{2} = 7225 m^2$$

$$\begin{aligned}
\text{Now length of each region} &= \text{Width of the field} \times 2 \\
&= \sqrt{7225} \\
&= 85m \\
&= 85 \times 2 = 170m \\
\text{Perimeter} &= 2(170 + 85) \\
&= 2(255) \\
&= 510m
\end{aligned}$$

Q3.   **(A)**

$$\begin{aligned}
\text{Cost} &= \text{Rs. } 5840.64 \\
&= 584064 \text{ paisas} \\
\text{Area} &= \frac{584064}{6} = 97344 \\
\text{Side} &= \sqrt{97344} \\
&= 312 \text{ m}
\end{aligned}$$

Q4.   **(C)**

```
1 | 1.00 06 02 19 (1.0003
    | 1
2.0003| 00 06 02 19
    |    6 00 09
    |       210
```

There are eight places after the decimal in the given number. so after subtracting .0000021 from th given number the remainder would be zero. So .0000021 is the required number.

Q5.   **(A)** $313 \times 313 = 97969$

Q6.   **(D)**   Side length $= 6cm$

$$\text{Sides of 256 square shapes} = \sqrt{256} = 16$$

$$\text{Size of the square is} = 16 \times 6 = 96$$

Q7.   **(D)**

$$\begin{aligned}
\text{Number of students in front row} &= \sqrt{9224 - 8} \\
&= \sqrt{9216} \\
&= 96
\end{aligned}$$

Q8.   **(B)**   The first two integers those square end with 9 are 3 and 7.

The difference between there is

$$49 - 9 = 40$$

Q9.   **(C)**   Suppose breadth $= x$

Length $= 10x$

$$\text{Area} = (10x) \times (x) = 75690$$

$\Rightarrow$ $\qquad 10x^2 = 75690$

$\Rightarrow$ $\qquad x^2 = 7569$

$\Rightarrow$ $\qquad x = 87 = \text{breadth}$

$\Rightarrow$ $\qquad 10x = 870 = \text{length}$

$\qquad$ Perimeter $= 2(\text{Length} + \text{breadth})$

$\qquad\qquad\qquad = 2(87 + 870)$

$\qquad\qquad\qquad = 1914 \text{ m}$

Q10. **(C)** Side of each square $= \sqrt{0.25} = 0.5\text{km}$

$\qquad$ Length of the railings $= (.5 + .5)2 = 2\text{km}$

$\qquad\qquad\qquad = 2 \times 1000 = 2000 \text{ m}$

$\qquad$ Cost of railing $= 2000 \times 101 = \text{Rs. } 202000$

Q11. **(B)** Size of the square sheet $= 2.1\text{m} = 2.1 \times 100 = 210\text{cm}$

$\qquad$ Area of square root $= 210 \times 210$

$\qquad\qquad\qquad = 44100 \text{ cm}^2$

So, No. of 1cm square pieces will be $= 1 \times 44100 = 44100$

Q12. **(C)** $\qquad$ Let the width $= b$, then length $= 2b$

$\qquad\qquad$ Area $= b \times 2b = 2b^2$

$\qquad$ Now $\qquad 2b^2 = 1152 \text{ (given)} \Rightarrow b^2 = 576$

$\Rightarrow$ $\qquad b = \sqrt{576} = 24$

$\qquad\qquad$ length $= 2b \Rightarrow \text{length} = 2 \times 24 = 48$

$\qquad$ Perimeter of the field $= 2(24 + 48)$

$\qquad\qquad\qquad = 2(72) = 144\text{m}$

Q13. **(D)** There is no way to simplify $\sqrt{x^2 + y^2}$.

Q14. **(C)** $\sqrt{\dfrac{a^2}{16} + \dfrac{a^2}{25}} = \sqrt{\dfrac{25a^2 + 16a^2}{400}} = \sqrt{\dfrac{41a^2}{400}} = \dfrac{a\sqrt{41}}{20}$

Q15. **(A)** $\sqrt{24a} \times \sqrt{6a}$

$\qquad = \sqrt{144a^2}$

$\qquad = 12a$

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