

## Chapter 5

## FRACTIONS AND DECIMALS

**FRACTIONS:**

If any unit be divided into any number of equal parts, one or more of these parts is called a fraction of the unit.

**Example:** The fractions one-fourth, two-third and three-fourth are respectively written as  $\frac{1}{4}$ ,  $\frac{2}{3}$  and  $\frac{3}{4}$

**NUMERATOR AND DENOMINATOR:**

The upper number, which shows the number of parts taken to form the fraction, is called numerator.

The lower number, which indicates the number of equal parts in which the unit is divided, is called denominator.

**Terms of The Fraction:**

The numerator and the denominator of a fractions are called its terms.

**Note:** A fraction is also called a rational number.

**Lowest Terms of a Fraction:**

When the numerator and the denominators of a fraction have no common factor, the fraction is said to be in its lowest terms:

$$\text{Example: } = \frac{6}{10} = \frac{3 \times 2}{5 \times 2}$$

In the above example denominator and the numerator have a common factor, thus  $\frac{6}{10}$  is not in its lowest terms. If we cancel out 2 by dividing numerator and denominator by 2 we find  $\frac{3}{5}$ , which has no common factor.

hence  $\frac{3}{5}$  is in its lowest terms.

**Proper Fraction:**

A proper fraction is one whose numerator is less than the denominator.

**Example:**  $\frac{2}{3}$ ,  $\frac{5}{7}$ ,  $\frac{23}{46}$  are proper fractions.

**Note:** The value of proper fractions is always less than 1

**IMPROPER FRACTION:**

A fraction whose numerator is equal to or greater than the denominator is called improper fraction.

**Example:**  $\frac{15}{13}$ ,  $\frac{13}{5}$ , and  $\frac{21}{14}$  are improper fractions.

**Note:** The value of an improper fractions is always more than as equal to 1.

**Mixed Fraction:**

When an improper fraction is changed to consist of a whole number and a fraction, it is called a mixed fraction.

**Example:** The improper fraction  $\frac{15}{13}$  can be written as  $\frac{2}{13}$ , which is a mixed fraction.

$$\begin{array}{r} 13 \overline{) 15} \\ \underline{13} \\ 2 \end{array}$$

**Compound Fraction:**

A fractions of a fraction is called a compound fraction.

**Example:**  $\frac{1}{3}$  of  $\frac{3}{5}$  is a compound fraction.

$$\text{Thus } \frac{1}{3} \text{ of } \frac{3}{5} = \frac{1}{3} \times \frac{3}{5} = \frac{1}{5}$$

**Complex Fractions:**

A complex fraction is one in which the numerator or denominator or both are fractions.

**Example:**  $\frac{3/2}{5}$ ,  $\frac{3}{2/5}$ ,  $\frac{2/5}{3/7}$  and  $\frac{1/3 + 1/2}{2/3 - 1/5}$  are complex fractions.

**Example 1:** One third of one-seventh of a plot is sold Rs. 45000. What is the value of six-twenty fifth plot.

**Solution:** One third of one seventh =  $\frac{1}{3} \times \frac{1}{7} = \frac{1}{21}$

Now,  $\frac{1}{21}$  of a plot costs = Rs. 45000.

$$\begin{aligned} \frac{6}{25} \text{ of the plot will cost} &= \frac{45000}{1/21} \times \frac{6}{25} \\ &= \frac{45000 \times 21 \times 6}{25} \end{aligned}$$

**Example 2:** A sum of money increased by its sixth part amount to Rs. 56. Find the sum.

**Solution:** Let x be the amount of money, thus

$$\begin{aligned} x + \frac{x}{6} = 56 &\Rightarrow \frac{6x + x}{6} = 56 \\ &\Rightarrow \frac{7x}{6} = 56 \\ &\Rightarrow 7x = 56 \times 6 \\ &\Rightarrow x = \frac{56 \times 6}{7} = 48 \end{aligned}$$

### VULGAR FRACTIONS

In questions of fractions signs +, -, ×, ÷, "of" ('of' signifies *multiplication*) and brackets are often involved. In simplifying these questions the following order must be followed:

#### IMPORTANT POINTS

- (i) Remove the brackets.
- (ii) Then quantities which are connected by 'of' should be simplified.
- (iii) Then division and multiplication are carried out.
- (iv) Operation of addition and subtraction are performed at last.

**Note:** The above rules can be easily remembered by the word 'BODISA' of which 'B' stands for bracket for 'of', D for division, I for into, S for subtraction and A for addition.

#### Model Examples:

**Example 3:**  $7\frac{1}{2} - \frac{1}{9} \left[ 3\frac{3}{4} \div \left\{ \frac{5}{6} \text{ of } \frac{2}{3} \left( \frac{1}{3} - \frac{1}{4} - \frac{1}{6} \right) \right\} \right]$

**Solution:**

$$\begin{aligned} &= 7\frac{1}{2} - \frac{1}{9} \left[ \frac{15}{4} + \left\{ \frac{5}{6} \text{ of } \frac{2}{3} \left( \frac{1}{3} - \frac{3-2}{12} \right) \right\} \right] \\ &= 7\frac{1}{2} - \frac{1}{9} \left[ \frac{15}{4} + \left\{ \frac{5}{6} \text{ of } \frac{2}{3} \left( \frac{1}{3} - \frac{1}{12} \right) \right\} \right] \\ &= 7\frac{1}{2} - \frac{1}{9} \left[ \frac{15}{4} + \left\{ \frac{5}{6} \text{ of } \frac{2}{3} \left( \frac{3}{12} \right) \right\} \right] \\ &= 7\frac{1}{2} - \frac{1}{9} \left[ \frac{15}{4} + \left\{ \frac{5}{6} \text{ of } \frac{1}{6} \right\} \right] \\ &= 7\frac{1}{2} - \frac{1}{9} \left[ \frac{15}{4} + \frac{5}{36} \right] \end{aligned}$$

$$= 7\frac{1}{2} - \frac{1}{9} \left[ \frac{15}{4} \times \frac{36}{5} \right]$$

$$= 7\frac{1}{2} - \frac{1}{9} \times 3 \times 9 = 7\frac{1}{2} - 3 = 4\frac{1}{2} \text{ Ans.}$$

**Example 4:** Simplify

$$\frac{1}{6} + \frac{5}{12} \times \left( \frac{4}{5} - \frac{5}{7} \right) \div \frac{1}{3} + \frac{1}{7} - \frac{2}{5}$$

*if  $\frac{2}{3} - \frac{3}{5}$  of  $\frac{4}{7}$*

**Solution:**

$$= \frac{1}{6} + \frac{5}{12} \times \left( \frac{4}{5} - \frac{5}{7} \right) \div \frac{1}{3} + \frac{1}{7} - \frac{2}{5}$$

$$= \frac{3}{4} \times \frac{5}{3} - \frac{3}{5} \times \frac{11}{7} \div \frac{1}{5} + \frac{1}{9} - \frac{2}{7}$$

$$= \frac{1}{6} + \frac{5}{12} \times \left( \frac{28-25}{35} \right) \div \frac{35+15-42}{105}$$

$$= \frac{5}{4} - \frac{33}{35} \div \frac{63+35-90}{315}$$

$$= \frac{1}{6} + \frac{1}{28} \div \frac{8}{105} \times \frac{315}{8}$$

$$= \frac{14+3}{84} \div 3 = \frac{17}{84} \times \frac{140}{43} \times \frac{1}{3}$$

$$= \frac{85}{387} \text{ Ans.}$$

**Continued Fraction:**

The fractions of the form  $a + \frac{b}{c + \frac{d}{e + \frac{f}{g}}}$

etc. are known as continued fractions where  $a, b, c, \dots$  etc., are any numbers.

**Note:** In order to simplify such fractions, we begin with the lowest part and proceed step by step, upwards.

**Model Examples:**

**Example 5:** Simplify:

$$\left\{ 1 + \frac{1}{2 + \frac{2}{3 + \frac{3}{4}}} \right\} \div \left\{ 4 + \frac{4}{3 + \frac{3}{2}} \right\}$$

**Solution:**

$$\left\{ 1 + \frac{1}{2 + \frac{2}{15}} \right\} \div \left\{ \frac{4}{4 + \frac{4}{9}} \right\}$$

$$= \left\{ 1 + \frac{1}{2 + \frac{8}{15}} \right\} \div \left\{ \frac{4}{4 + \frac{8}{9}} \right\}$$

$$= 1 + \left\{ \frac{1}{\frac{38}{15}} \right\} \div \left\{ \frac{4}{\frac{44}{9}} \right\}$$

$$= \left\{ 1 + \frac{15}{38} \right\} \div \left\{ \frac{36}{44} \right\}$$

$$= \frac{53}{38} \times \frac{44}{36} = \frac{583}{342}$$

**Ans.**

### DECIMAL FRACTION

A fraction involving decimal point is called decimal fraction.

**Conversion of a decimal fraction into vulgar fraction:**

**Rule.** Write down the given number in the numerator omitting the decimal point and for the denominator write 1 followed by as many zeroes as there are figures on the right of the decimal point.

$$\text{As } 46.76 = \frac{4676}{100}$$

$$\text{and } 199.0083 = \frac{1990083}{10000}$$

### Model Example

Q1. Simplify

$$\frac{0.1 \times 0.1 \times 0.1 + 0.01 + 0.01 \times 0.01}{0.2 \times 0.2 \times 0.2 + 0.08 + 0.04 \times 0.02}$$

**Solution:**

$$\begin{aligned} &= \frac{\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} + \frac{1}{100} + \frac{1}{100} \times \frac{1}{100}}{\frac{2}{10} \times \frac{2}{10} \times \frac{2}{10} + \frac{8}{100} + \frac{4}{100} \times \frac{2}{100}} \\ &= \frac{\frac{1}{1000} + \frac{1}{100} + \frac{1}{10000}}{\frac{8}{1000} + \frac{8}{100} + \frac{8}{10000}} \end{aligned}$$

$$\begin{aligned}
 &= \left( \frac{1}{1000} + \frac{1}{100} + \frac{1}{10000} \right) \\
 &= \frac{\left( \frac{1}{1000} + \frac{1}{100} + \frac{1}{10000} \right)}{8 \left( \frac{1}{1000} + \frac{1}{100} + \frac{1}{10000} \right)} \\
 &= \frac{1}{8} \quad \text{Ans.}
 \end{aligned}$$

### Multiple Choice Questions (MCQs)

- Q1. If  $\frac{5}{x}$ ,  $\frac{8}{x}$ , and  $\frac{13}{x}$  are all in lowest terms. Then how many integers,  $x$ , between 30 and 40?
- (A) 5 (B) 1  
(C) 2 (D) 3  
(D) None of these
- Q2.  $\frac{6}{6} \times \frac{6}{12} \times \frac{6}{18} \times \frac{6}{24} \times \frac{6}{30}$  equals:
- (A)  $\frac{1}{120}$  (B)  $\frac{1}{2}$   
(C)  $\frac{1}{30}$  (D) 1  
(D) None of these
- Q3. If  $\frac{4}{13}$  of a number is 39, what is  $\frac{8}{13}$  of that number?
- (A)  $\frac{39}{4}$  (B) 78  
(C) 16 (D)  $\frac{39}{8}$
- Q4.  $\frac{3}{4}$  of 28 is equal to  $\frac{30}{7}$  of what number?
- (A) 90 (B) 45  
(C) 30 (D) 56  
(D) None of these
- Q5. Which of the following is less than  $\frac{5}{11}$ ?
- (A)  $\frac{3}{2}$  (B)  $\frac{2}{3}$   
(C)  $\frac{1}{2}$  (D)  $\frac{2}{5}$   
(D) None of these
- Q6. There are 20 boys in a class. Five of them are left-handed. What fraction of the class is left handed?
- (A)  $\frac{1}{5}$  (B)  $\frac{1}{2}$   
(C)  $\frac{1}{4}$  (D)  $\frac{2}{11}$

- Q7. A chemical solution contains 8% of acid. If there is 15ml of acid, what is the volume of the solution?
- (A) 125.5 mL (B) 187.5 mL  
(C) 225.5 mL (D) 171.5 mL
- Q8. What fractional part of a week is 98 hours?
- (A)  $\frac{7}{98}$  (B)  $\frac{7}{12}$   
(C)  $\frac{1}{20}$  (D)  $\frac{1}{7}$
- Q9. A village has 5860 voters, of whom 7% usually forget to vote. In order to win an election, a candidate must gain at least 50% of the remaining votes. How many votes does he need in order to win?
- (A) 2725 (B) 410  
(C) 5450 (D) None of these
- Q10. What fraction is exactly midway between  $\frac{1}{3}$  and  $\frac{1}{4}$ ?
- (A)  $\frac{7}{12}$  (B)  $\frac{7}{24}$   
(C)  $\frac{29}{11}$  (D)  $\frac{1}{2}$
- Q11.  $\frac{4}{9}$  of a number is 12. What is the number?
- (A) 27 (B) 36  
(C) 18 (D) 16
- Q12. Ali purchased some goldfish. During the first week,  $\frac{1}{5}$  of them died, and during the second week,  $\frac{3}{8}$  of those still alive at the end of the first week died. What is the fraction of the original goldfish still alive after two weeks?
- (A)  $\frac{1}{2}$  (B)  $\frac{3}{2}$   
(C)  $\frac{5}{2}$  (D)  $\frac{4}{3}$
- Q13.  $\frac{3}{8}$  of a number is 10. What is the number?
- (A) 91 (B) 81  
(C) 23 (D) 27
- Q14.  $\frac{5}{8}$  of 24 is equal to  $\frac{15}{7}$  of what number?
- (A) 15 (B) 105  
(C) 35 (D) 7
- Q15. A German class has 12 boys and 18 girls. What is the fraction of the class boys?
- (A)  $\frac{1}{6}$  (B)  $\frac{3}{5}$   
(C)  $\frac{2}{3}$  (D)  $\frac{4}{15}$

## Explanatory Answers

Q1. (D) If  $x$  is even, then  $\frac{8}{x}$  will not be in lowest term. This is because, both  $x$  and 8 are divisible by 2.

Now we take the odd number between 30 and 40, these are; 31, 33, 35, 37, 39. In these numbers, we see that 35 and 39 are divisible by 5 and 13, respectively. Thus only 31, 33 and 37 are required numbers.

Q2. (A) Simplifying  $\frac{6}{6} \times \frac{6}{12} \times \frac{6}{18} \times \frac{6}{24} \times \frac{6}{30}$

$$\frac{1}{1} \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$$

Q3. (B) As  $\frac{4}{13}$  of a number is 39. Therefore the  $\frac{8}{13}$  of that number will be 78

Because  $\frac{8}{13} = \frac{4}{13} \times 2$ , and  $\frac{4}{13}$  of a number is 39, therefore double of  $\frac{4}{13} \left( \frac{4}{13} \times 2 = \frac{8}{13} \right)$  should be equal to  $39 \times 2 = 78$ .

Q4. (A) Let  $x$  be the required number, then by given condition

$$28 \div \frac{4}{3} = x \div \frac{30}{7}$$

$$28 \times \frac{3}{4} = x \times \frac{7}{30}$$

$$21 = x \times \frac{7}{30}$$

$$\frac{21 \times 30}{7} = x$$

$$\Rightarrow \boxed{x = 90}$$

Q6. (C) Left handed = 5

$$\text{Total} = 20$$

$$\text{So, fraction} = \frac{5}{20} = \frac{1}{4}$$

Q7. (B) 8mL acid in solution = 100mL

$$1\text{mL acid in solution} = \frac{100}{8} = 12.5 \text{ mL}$$

$$15\text{mL acid in solution} = 12.5 \times 15 \\ = 187.5 \text{ mL}$$

Q8. (B) There are 7 days in a week, and each day has 24 hours. Therefore, Hours in a week =  $24 \times 7 = 168$

$$\text{The required fraction is: } \frac{98}{168} = \frac{7}{12}$$

Q9. (A) People does not give vote =  $\frac{7}{100} \times 5860$

$$= 7 \times 58.6$$

$$= 410.2$$

$$\text{People does not give vote} \cong 410 \text{ people}$$

$$\begin{aligned}\text{Remaining people} &= 5860 - 410 \\ &= 5450 \text{ people}\end{aligned}$$

$$\begin{aligned}\text{Candidate must gain vote} &= 5450 \times \frac{50}{100} \\ &= 2725 \text{ vote}\end{aligned}$$

Q10. (B) The midway fraction of the fractions  $\frac{1}{3}$  and  $\frac{1}{4} = \frac{1}{2} \left( \frac{1}{3} + \frac{1}{4} \right) = \frac{1}{2} \left( \frac{7}{12} \right) = \frac{7}{24}$

Q11. (A) Let the required number be "x", then according to given condition  $\frac{4}{9} \times x = 12 \Rightarrow x = \frac{12}{\frac{4}{9}}$

$$= 12 \times \frac{9}{4} = 27$$

Q12. (A) Let the number of fish purchased = x

$$\text{During first week (died fish)} = \frac{1}{5} \times x = \frac{x}{5}$$

$$\text{Still alive} = x - \frac{1}{5}x = \frac{4}{5}x$$

$$\text{During second week (died fish)} = \frac{4}{5}x \times \frac{3}{8} = \frac{3}{10}x$$

$$\text{Fish at the end of two weeks} = \frac{4x}{5} - \frac{3x}{10} = \frac{8x - 3x}{10} = \frac{5x}{10} = \frac{1}{2}x$$

$$\text{So fraction} = \frac{\frac{1}{2}x}{x} = \frac{1}{2}$$

Q13. (D) Let the number = x

$$\text{Then } \frac{3}{8} \times x = 10$$

$$\Rightarrow x = \frac{80}{3}$$

$$\Rightarrow x = 26.67 = \boxed{27}$$

Q14. (D) Let the number = x

$$\text{Then } \frac{15}{7} \times x = \frac{5}{8} \times 24$$

$$\Rightarrow \frac{15 \times x}{7} = 15$$

$$\Rightarrow x = \frac{7 \times 15}{15} = \boxed{7}$$

Q15. (D) No. of boys = 12

$$\text{No. of girls} = 18$$

$$\text{Total} = 12 + 18 = 30$$

$$\text{Required fraction} = \frac{12}{30} = \frac{4}{15}$$

\*\*\*\*\*