NUMERICAL PROBLEMS

P.7.1 A 100 g body hung on a spring elongates the spring by 4 cm. When a certain object is hung on the spring and set vibrating its period is 0.568 s. What is the mass of the object pulling the spring?

DATA: Mass of the body $= m = 100\,g = 0.1\,kg$

Extension in the spring $= x = 4\,cm = 0.04\,m$

Time period $= T = 0.568\,s$

Mass of the object $= m' = ?$

Sol. According to Hook's law,

$F = kx$ or $k = \frac{F}{x}$ —— (1)

$k = \frac{mg}{x} = \frac{0.1 \times 9.8}{0.04} = 24.5\,Nm^{-1}$ —— (2) $\because F = W = mg$

Now for mass spring system, we know that

$$T = 2\pi\sqrt{\frac{m'}{k}}$$

or $$T^2 = 4\pi^2 \cdot \frac{m'}{k}$$

$$\Rightarrow \quad m' = \frac{T^2 k}{4\pi^2}$$

$$m' = \frac{(0.568)^2 \times 24.5}{4 \times (3.14)^2} = \boxed{0.2\,kg} \text{ or } \boxed{200\,g}$$

P.7.2 A load of 15 g elongates a spring by 2 cm. If body of mass 294 g is attached to the spring and is set into vibration with an amplitude of 10 cm, what will be its (i) period (ii) spring constant (iii) max. speed of its vibration?

DATA: Load $= m = 15\,g = 0.015\,kg$

Extension in the spring $= x = 2\,cm = 0.02\,m$

Mass attached to the spring $= m' = 294\,g = 0.294\,kg$

Amplitude $= x_0 = 10\,cm = 0.1\,m$

(i) Time period $= T = ?$

(ii) Spring constant $= k = ?$

(iii) Max. Speed $= v_0 = ?$

(P.T.O)

Sol. (i) According to Hook's law,

$$F = Kx \quad \text{——— (1)}$$

But, $F = W = mg$ ——— (2)

we have $Kx = mg$

so $K = \frac{mg}{x} = \frac{0.015 \times 9.8}{0.02} = \boxed{7.35\ N\,m^{-1}}$

(ii) Time period of mass spring system is

$$T = 2\pi\sqrt{\frac{m}{K}} \quad \text{——— (3)}$$

$$T = 2 \times 3.14 \times \sqrt{\frac{0.294}{7.35}} = \boxed{1.26\ S}$$

(iii) We know that

$$V_o = x_o\sqrt{\frac{K}{m''}} \quad \text{——— (4)}$$

In this case m'' = Load + mass

$= 0.015 + 0.29 = 0.309\ kg$ ——— (5)

Putting the values, we get

$$V_o = 0.1 \times \sqrt{\frac{7.35}{0.309}} = 0.488\ m\,s^{-1} \simeq \boxed{48.8\ cm\,s^{-1}}$$

P.7.3 An 8 kg body executes SHM with amplitude 30 cm. The restoring force is 60 N when the displacement is 30 cm. Find (i) Period (ii) Acceleration, speed, K.E and P.E when the displacement is 12 cm.

DATA. Mass of body $= m = 8.0$ kg

Amplitude $= x_o = 30$ cm $= 0.3$ m

Restoring force $= F = 60$ N

Displacement $= x = 30$ cm $= 0.3$ m

(i) Period $= T = ?$

(ii) Acceleration $= a = ?$

(iii) speed $= V = ?$

(iv) K.E $= ?$

(v) P.E $= ?$ (When the displacement, $x = 12$ cm)

Sol. (i) According to Hook's law,

$$F = Kx$$

$$K = \frac{F}{x} = \frac{60}{0.3} = 200\ N\,m^{-1} \quad \text{——— (1)}$$

(P.T.O)

Using the formula for time period of spring mass system,

$$T = 2\pi\sqrt{\frac{m}{K}} \quad \text{—— (2)}$$

$$T = 2\times 3.14\times\sqrt{\frac{8}{200}} = \boxed{1.3\ s}$$

(ii) As we know

$$F = -Kx \quad \text{—— (3)}$$

and $F = ma$ —— (4)

or $ma = -Kx$

$$a = -\frac{K}{m}\times x \quad \text{—— (5)} \qquad \because x = 12cm = 0.12\ m$$

$$a = -\frac{200}{8}\times 0.12 = \boxed{-3\ ms^{-2}}$$

Negative sign indicates that acc. is directed towards the mean point.

(iii) The speed of any body executing SHM is given by

$$v = \omega\sqrt{x_0^2 - x^2} \quad \text{—— (6)}$$

But $\omega = \frac{2\pi}{T} = \frac{2\times 3.14}{1.3} = 4.82\ Hz$ —— (7)

$\therefore$ $V = 4.82\sqrt{(0.3)^2 - (0.12)^2}$

$$V = \boxed{1.33\ ms^{-1}}$$

(iv) For K.E, we know that

$$K.E = \frac{1}{2}\times Kx_0^2\left(1 - \frac{x^2}{x_0^2}\right) \quad \text{—— (8)}$$

$$= \frac{1}{2}\times 200\times 0.3\left(1 - \frac{(0.12)^2}{(0.3)^2}\right)$$

$$K.E = 7.56\ J \simeq \boxed{7.6\ J}$$

(v) For P.E, we know that

$$P.E = \frac{1}{2}\times Kx^2 \quad \text{—— (9)}$$

$$= \frac{1}{2}\times 200\times(0.12)^2$$

$$= \boxed{1.44\ J}$$

(P.T.O)

P.7.4 A block of mass 4.0 kg is dropped from a height of 0.8 m on to a spring of spring constant $K = 1960\ N\ m^{-1}$. Find the max. distance through which the spring will be compressed.

DATA. Mass of the block = m = 4 kg

Height = h = 0.8 m

Spring constant = K = 1960 $N\ m^{-1}$

~~Max. distance = x_0 = ?~~

Sol. We know that

$$P.E = mgh \quad \text{--- (1)}$$

$$P.E = 4 \times 9.8 \times 0.8 = 31.36\ J \quad \text{--- (2)}$$

When the block is dropped on the spring, the spring will be compressed through max. distance x_0 due to the P.E of the block. Thus

$$P.E = \frac{1}{2} \times K x_0^2 \quad \text{--- (3)}$$

$$x_0^2 = \frac{2\,P.E}{K}$$

$$x_0^2 = \frac{2 \times 31.36}{1960} = 0.18\ m^2$$

$$x_0 = \sqrt{0.18} = \boxed{0.42\,m}$$

P.7.5 A simple pendulum is 50 cm long. What will be its frq. of vibration at a place where $g = 9.8\ m s^{-2}$?

DATA. Length of simple pendulum = l = 50 cm = 0.5 m

Acc. due to gravity = g = 9.8 $m s^{-2}$

Freq. of simple pendulum = f = ?

Sol. Time period of simple pendulum is;

$$T = 2\pi\sqrt{l/g}$$

$$T = 2 \times 3.14 \times \sqrt{\frac{0.5}{9.8}} = 1.41\ s \quad \text{--- (1)}$$

We know that

$$f = \frac{1}{T}$$

$$f = \frac{1}{1.41} = \boxed{0.71\ Hz}$$

(P.T.O)

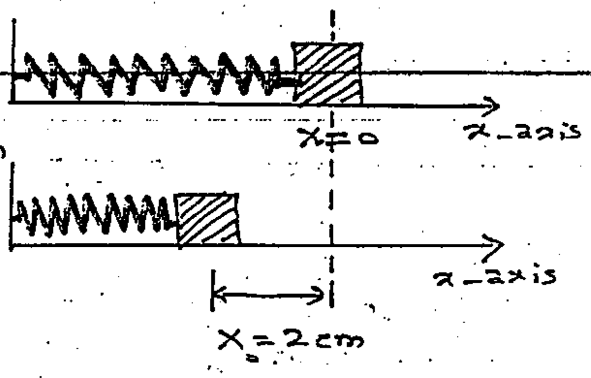
P.7.6 A block of mass 1.6 kg is attach to a spring with spring constant 1000 Nm^{-1} as shown. The spring is compressed through a distance of 2cm and the block is released from rest. Calculate the velocity of the block as it passes through the equilibrium position, $x = 0$ if the surface is friction less.

DATA. Mass of block $= m = 1.6$ kg

Spring constant $= K = 1000\ Nm^{-1}$

Max. displacement $= x_o = 2cm = 0.02m$

vel. at mean position $= V = ?$



Sol. As the velocity is max. at the mean position, so the formula is given by:

$$V = x_o \sqrt{\frac{K}{m}}$$

Putting the values,

$$V = 0.02 \times \sqrt{\frac{1000}{1.6}} = \boxed{0.5\ ms^{-1}}$$

P.7.7 A car of mass 1300 kg is constructed using a frame supported by four springs. Each spring has a spring constant 20,000 Nm^{-1}. If two people riding in the car have a combined mass of 160 kg, find the frq. of vibration of the car, when it is driven over a pot hole in the road. Assume the weight is evenly distributed.

DATA. Mass of car $= m_1 = 1300$ kg

Mass of two people $= m_2 = 160$ kg

Spring constant of each spring $= K = 20,000\ Nm^{-1}$

Frequency of vibration $= f = ?$

Sol. The formula for the time period of mass spring system is, $T = 2\pi\sqrt{\frac{m}{K}}$ But $f = \frac{1}{T}$ so

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \quad \text{——— (1)}$$

$$f = \frac{1}{2\times 3.14} \times \sqrt{\frac{80,000}{1460}}$$

Hence $f = \boxed{1.18\ Hz}$

∴ Total mass $= m$
$= m_1 + m_2$
$= 1300 + 160$
$= 1460$ kg

$K = 4 \times 20,000$
$= 80,000\ Nm^{-1}$

(P.T.O)

P.7.8 Find the amplitude, frq. and period of an object vibrating at the end of a spring, if the eq. for its position, as a function of time is $x = 0.25 \cos(\frac{\pi}{8} t)$. What is the displacement of the object after 2.0 s?

DATA. Time $= t = 2.0$ s

(i) Amplitude $= x_0 = ?$ (ii) Frequency $= f = ?$

(iii) Period $= T = ?$ (iv) Displacement $= x = ?$

Sol. (i) The general eq. of SHM is

$$x = x_0 \cos \omega t \quad \text{(1)}$$

comparing the given eq.

$$x = 0.25 \cos\left(\frac{\pi}{8} t\right) \quad \text{(2)}$$

Amplitude $= x_0 = \boxed{0.25\ m}$

(ii) $\omega = \pi/8$

But $\omega = 2\pi f$

$$\therefore f = \frac{\omega}{2\pi} = \frac{\pi/8}{2\pi} = \boxed{\frac{1}{16}\ Hz}$$

(iii) The time period is given as

$$T = \frac{1}{f} = \frac{1}{1/16} = \boxed{16\ s}$$

(iv) Displacement after 2 seconds:

$$x = 0.25 \cos\left(\frac{\pi}{8} \times t\right)$$

~~$x = 0.25 \cos\left(\frac{\pi}{8} \times 2\right)$~~

$$x = \boxed{0.18\ m}$$

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